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Phase-shifting method for two-dimensional birefringence measurement with return-path beams

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ABSTRACT

The paper describes the phase-shifting method for measuring two dimensional birefringence distributions with return-path polarimeter scheme. Eight or sixteen images in polarized light are processed for determining of specimen's retardance and azimuth distributions. The principal formulas that describe the mathematical processing are presented. The method allows us to find the image of specimen's birefringence with nonuniform distribution of fast axis azimuth and retardance. The method gives the exact solution for any retardance value. Approximation equations for determining of small retardance are presented. The measurement method is highly effective for research of vector or tensor physical fields, which are accompanied by birefringence. For example, inner stresses, electrical and magnetic fields, heat flows, birefringence liquid flows et al.

Keywords: polarization, birefringence measurement, return-path polarimeter, and phase shifting technique

1.INTRODUCTION

The two-dimensional polarimeters make possible to get the polarization images of optical specimen under test with nonuniform birefringence field. These images are named polarograms and they show distribution of retardance and principal axes azimuth in the specimen. At first the polarimeters were applied for investigation of inner stress in construction models from optical transparence material with use of photoelasticity.[1] Improvement of two dimensional polarimeters precision has allowed to expand the class of researched problems. In particular, there are investigations of biological specimens[2-4] and optical disks,[5] human cornea,[6] checking quality of optical elements,[7] substrates from GaAs and Si,[8] substrates from $LiNbO_3$,[9] electrooptic crystals,[10] study distributions of heat flows,[11] electric[12] or magnetic fields[13] etc.

The article describes the two-dimensional return-path polarimeter based on the compensation birefringence detector.[14] Owing to superposition of a probe beam and return beam we have the following advantages:

a) due to double passing of the beam through a specimen threshold of sensitivity gets lower;

b) decrease in the number of optical elements simplifies the device and makes it cheaper;

c) if the specimen under test is disposed in a hermetically isolated cell to entrance and exit of radiation, the same window is used;

d) remote control can be realized if direct access to specimen is difficult;

e) it is possible to investigate spherical and cylindrical lenses without a liquid immersion.

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2.DESCRIPTION OF the POLARIMETER

Optical configuration of the polarimeter is shown in Fig.1. The device works in the following manner. A linearly polarized laser beam is launched in a polarization maintain optical fiber PMF. Output fiber tip is fixed on the first rotating body RB1. Additionally, a collimating lens and a polarization beamsplitter cube PBS are mounted on the first body. The main fiber polarization axis is perpendicular to the output beam incidence plane on the cube's splitter surface. Therefore, the beam reflects from the cube completely. Here we receive a probe beam. The body RB1 can rotate around the probe beam's axis. The azimuth of the first body determines initial azimuth of the probe beam's polarization plane. Then the probe beam passes through a quarter wave plate QWP which is mounted in the second rotating body RB2. The plate QWP can turn around the probe beam's axis also. The probe beam reflects from the mirror and passes through the sample two times. If a sample is a spherical or cylindrical lens, the mirror has the same spherical shape accordingly. A concave mirror is more suitable for this purpose. The return beam passes back through the quarter wave plate and falls on the beamsplitter cube. The parallel-polarized part of the beam passes through the cube and creates the mirror image on the CCD camera surface with use of a camera lens. Because of small distance between the sample and the mirror we actually have the sample image on the camera surface. Diaphragm D decreases the image noise.

For experimental testing we used: He-Ne laser (λ=633nm, short term stability of output power less than 0.02%), polarization maintain single mode fiber "Newport Corp." (NA=0.16, extinction ratio more than 1000:1), polarization beamsplitter cube "Spindler & Hoyer Corp." (rib length 20 mm, extinction ratio more than 10000:1), zero order quarter wave plate "Tower Optical Corp." (diameter 25.4 mm, tolerance λ/500), CCD camera "Hamamatsu Corp." (10 bits of gray levels). Thus the setup allows us to study a sample with diameter up to 18 mm. Sensitivity threshold of the polarimeter comes to 0.4° or 0.7 nm at wavelength 633 nm...

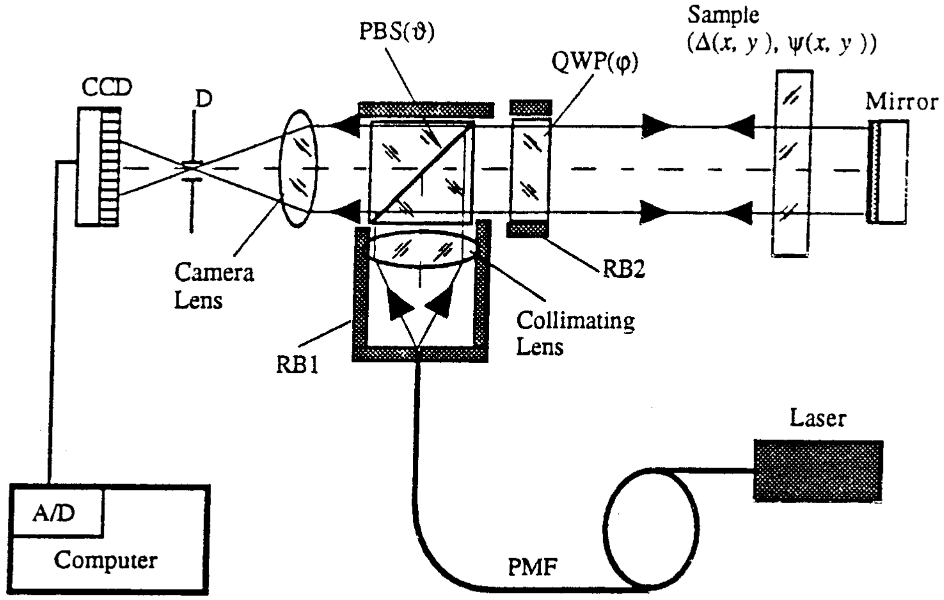

Fig.1 Experimental set up for two dimensional birefringence measurements. PBS (θ) polarization beamsplitter cube with azimuth θ; QWP (φ) quarter wave plate with the fast axis azimuth φ; Δ(x,y), ψ(x,y): retardance and azimuth distributions of the specimen; PMF: polarization maintain fiber; RB1 and RB2: the first and the second rotating bodies; CCD: CCD camera; D: diaphragm (pin hole).

3.MEASUREMENT OF TWO DIMENSIONAL BIREFRINGENCE DISTRIBUTION

Intensity distribution on CCD camera's area $I(x,y)$ depends upon intensity distribution of probe beam $I_0(x,y)$ and distribution of birefringence parameters $\Delta(x,y)$ and $\psi(x,y)$.[15] Here x,y are coordinate axes, $\Delta(x,y)$ is retardance distribution of sample, $\psi(x,y)$ is the fast azimuth distribution of a sample. There are several ways for determining of sample's retardance $\Delta(x,y)$ and azimuth $\psi(x,y)$ distribution. For instance, it is possible to compare the phase shift of function $I_n(x,y,\theta)$ at two or four value of parameter φ_n. Therefore the method is named "phase shifting method". The first case can be named as two-step method and the second case can be named as four-step method accordingly.

16 or 8 image pictures on CCD camera are registered for the following azimuth of the PBS θ and the fast axis azimuth of the quarter waveplate φ:

$$\begin{aligned} \varphi_n &= 45°(n-1) \\ \theta_{nm} &= 22{,}5°(2n-m-1), \end{aligned} \tag{1}$$

where n equals 1, 2 and m equals from 1 to 4 in the first case, n and m are integer from 1 to 4 in the second case. Hereupon all the angle quantities are written as degrees.

For simplicity let us adduce vector and matrix of the corresponding azimuth angles of the quarter wave plate $\{\varphi_n\}$ and the polarization beamsplitter cube $\{\theta_{nm}\}$

$$\{\varphi_n\} = \begin{pmatrix} 0° \\ 45° \\ 90° \\ 135° \end{pmatrix} \text{ and } \{\theta_{nm}\} = \begin{pmatrix} 0° & -22.5° & -45° & -67.5° \\ 45° & 22.5° & 0° & -22.5° \\ 90° & 67.5° & 45° & 22.5° \\ 135° & 112.5° & 90° & 67.5° \end{pmatrix}.$$

At these angles we found the intensity distribution on CCD camera $\{I_{nm}(x,y)\}$ that will be the next:

$$I_{nm}(x,y) = \begin{pmatrix} \sin^2\Delta\sin^2 2\psi & \frac{1}{2}(\cos\Delta+\sin\Delta\sin 2\psi)^2 & \cos^2\Delta & \frac{1}{2}(\cos\Delta-\sin\Delta\sin 2\psi)^2 \\ \sin^2\Delta\cos^2 2\psi & \frac{1}{2}(\cos\Delta-\sin\Delta\cos 2\psi)^2 & \cos^2\Delta & \frac{1}{2}(\cos\Delta+\sin\Delta\cos 2\psi)^2 \\ \sin^2\Delta\sin^2 2\psi & \frac{1}{2}(\cos\Delta-\sin\Delta\sin 2\psi)^2 & \cos^2\Delta & \frac{1}{2}(\cos\Delta+\sin\Delta\sin 2\psi)^2 \\ \sin^2\Delta\cos^2 2\psi & \frac{1}{2}(\cos\Delta+\sin\Delta\cos 2\psi)^2 & \cos^2\Delta & \frac{1}{2}(\cos\Delta-\sin\Delta\cos 2\psi)^2 \end{pmatrix}. \tag{2}$$

Here for brevity we omitted the constant multiplier $I_0(x,y)$ and reflection coefficient of the mirror.

For sample with small retardance we can use the linear approximation:

$$I_{nm}(x,y)=\begin{pmatrix} 0 & \frac{1}{2}+\frac{\pi}{180^\circ}\Delta\sin 2\psi & 1 & \frac{1}{2}-\frac{\pi}{180^\circ}\Delta\sin 2\psi \\ 0 & \frac{1}{2}-\frac{\pi}{180^\circ}\Delta\cos 2\psi & 1 & \frac{1}{2}+\frac{\pi}{180^\circ}\Delta\cos 2\psi \\ 0 & \frac{1}{2}-\frac{\pi}{180^\circ}\Delta\sin 2\psi & 1 & \frac{1}{2}+\frac{\pi}{180^\circ}\Delta\sin 2\psi \\ 0 & \frac{1}{2}+\frac{\pi}{180^\circ}\Delta\cos 2\psi & 1 & \frac{1}{2}-\frac{\pi}{180^\circ}\Delta\cos 2\psi \end{pmatrix}. \tag{3}$$

It is necessary to note that every element $I_{nm}(x,y)$ is two dimensional intensity distributions for perfect polarimeter without any error.

These data are processed in the following way. At the beginning a vector $\vec{\Phi}$ is found where vector's components are determined as:

$$\Phi_n = \frac{I_{n4}-I_{n2}}{I_{n1}-I_{n3}}. \tag{4}$$

1) Linear approximation

According to matrix (3) we obtain:

$$\Phi_n = \frac{\pi}{90^\circ}\Delta\cos 2(\psi-45^\circ n). \tag{5}$$

In this case the sample retardance and fast axis azimuth are calculated using the next formulas:

a) for two-step method:

$$\Delta = \frac{90^0}{\pi}\sqrt{\Phi_1{}^2+\Phi_2{}^2}\ ; \qquad \psi = -\frac{1}{2}\cdot\tan^{-1}\frac{\Phi_1}{\Phi_2}\ ; \tag{6}$$

b) for four-step method:

$$\Delta = \frac{45^0}{\pi}\sqrt{(\Phi_1-\Phi_3)^2+(\Phi_4-\Phi_2)^2}\ ; \qquad \psi = \frac{1}{2}\cdot\tan^{-1}\frac{\Phi_1-\Phi_3}{\Phi_4-\Phi_2}. \tag{7}$$

The third term approximation errors for both methods equal between then and are the following:

$$\xi_\Delta = \left(\frac{\pi}{180^0}\right)^2\cdot\frac{8-3\sin^2 4\psi}{6}\cdot\Delta^3 \quad \text{and} \quad \zeta_\psi = -\frac{\pi}{180^0}\cdot\frac{\Delta^2}{8}\cdot\sin 8\psi, \tag{8}$$

where the retardance error is marked as ξ_Δ and the azimuth error is noted as ζ_ψ. In particular, if Δ is less than 15° the error ξ_Δ don't exceed 10%. At this point the azimuth error ζ_ψ is less than 0.5°.

2) Approximation with use of a tan− function

Let us develop the matrix elements (2) as a series in functions $\frac{1}{k}\tan(k\Delta)$, where k is a parameter. As can we show the optimal value of parameter k equals 2. In this case the sample retardance will be defined by the following formulas:

a) for two-steps method:

$$\Delta = \frac{1}{2}\tan^{-1}\sqrt{\Phi_1^{\,2} + \Phi_2^{\,2}}\ ; \tag{9}$$

b) for four-steps method:

$$\Delta = \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\sqrt{(\Phi_1 - \Phi_3)^2 + (\Phi_4 - \Phi_2)^2}\right). \tag{10}$$

The sample azimuth is determined by the formula (7) or (8).

The third term approximation errors of retardance ξ_Δ for both methods are the next:

$$\xi_\Delta = \frac{1}{2}\left(\frac{\pi}{180^0}\right)^2 \cdot \Delta^3 \sin^2 4\psi . \tag{11}$$

The approximation retardance error doesn't exceed 10% for Δ less 30°. As you can see tan− function approximation gives us more precise result.

3) Precise processing formulas

If the retardance is not small, we find a vector $\vec{X}$ with the next components:

$$X_n = \frac{\Phi_n}{1 + \sqrt{1 + \Phi_n^2}} \quad \text{for} \quad I_{n1} - I_{n3} < 0,$$

or

$$X_n = -\frac{\Phi_n}{1 + \sqrt{1 + \Phi_n^2}} \quad \text{for} \quad I_{n1} - I_{n3} > 0,$$

or

$$X_n = 1 \quad \text{for} \quad I_{n1} - I_{n3} = 0. \tag{12}$$

Where

$$X_1 = -X_3 = \tan\Delta \cdot \sin 2\psi ,$$

$$X_2 = -X_4 = -\tan\Delta \cdot \cos 2\psi . \tag{13}$$

From here we are able to get the precise solution for sample's parameters Δ and ψ.

a) For the two-step method the following formulas are used:

$$\Delta = \tan^{-1}\left(\sqrt{X_1^{\,2} + X_2^{\,2}}\right) \text{ and}$$

$$\psi = -\frac{1}{2}\cdot\tan^{-1}\left(\frac{X_1}{X_2}\right) \tag{14}$$

when $X_2 < 0$;

$$\psi = \pm 45^\circ$$

when $X_2 = 0$, and besides a sign "-" is taken if $X_1 < 0$ and a sign "+" is putted for $X_1 > 0$;

$$\psi = -\frac{1}{2} \cdot \tan^{-1}\left(\frac{X_1}{X_2}\right) \pm 90^\circ$$

when $X_2 > 0$, and besides a sign "-" is taken if $X_1 < 0$ and a sign "+" is putted for $X_1 \geq 0$.

b) for the four-step method the following formulas are used:

$$\Delta = \tan^{-1}\left(\frac{1}{2} \cdot \sqrt{(X_1 - X_3)^2 + (X_4 - X_2)^2}\right),$$

$$\psi = \frac{1}{2} \cdot \tan^{-1}\left(\frac{X_1 - X_3}{X_4 - X_2}\right)$$

when $X_4 - X_2 > 0$;

$$\psi = \pm 45^0$$

when $X_4 - X_2 = 0$, and besides a sign "-" is taken if $X_1 - X_3 < 0$ and a sign "+" is putted for $X_1 - X_3 > 0$;

$$\psi = \frac{1}{2} \cdot \tan^{-1}\left(\frac{X_1 - X_3}{X_4 - X_2}\right) \pm 90^0$$

when $X_4 - X_2 < 0$, and besides a sign "-" is taken if $X_1 - X_3 < 0$ and a sign "+" is putted for $X_1 - X_3 \geq 0$.

4.CONCLUSION

The phase-shifting methods allow to increase sensitivity of measurements and to remove errors caused by dark current and background. The methods diminish the influence of polarization beamsplitter imperfection essentially.

The four-step phase-shifting method has significantly smaller errors evoked by imperfection of a quarter waveplate, a radiation noise, a discreteness of gray levels and adjustment errors comparing to the two-step method.

The processing formulas are chosen for the specific conditions. If the retardance is small it's better to use the linear or tan-approximation. For large retardance we need the precise processing formulas. But in the last case it's necessary to use more powerful computer or to spend more time for mathematical processing.

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