Multiple Element Limitation (MEL) model version V equations:

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| **Mass Balance Equations:** | | | |
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| 15 | for *i* = *C*, *N*, or *P* | | |
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| **Process and Allometric Equations:** | | | |
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| 34 |  | 35 |  |
| 36 |  | 37 | for *i* = *C*, *N*, or *P* |
| 38 | for *i* = *C*, *N*, or *P* | 39 |  |
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Symbol definitions, values, and sources:

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| **State Variables** | | | | | |
| Eq. | Symbol | Name | Value | Units | Comments |
| 1 | *BC* | Vegetation C | 12006 | g C m-2 | Rastetter et al. (2013) |
| 2 | *BN* | Vegetation N | 80.64 | g N m-2 | Rastetter et al. (2013) |
| 3 | *BP* | Vegetation P | 10.940 | g P m-2 | Rastetter et al. (2013) |
| 4 | *DCWC* | Coarse woody debris C | 1313 | g C m-2 | Rastetter et al. (2013) |
| 5 | *DCWN* | Coarse woody debris C | 5.8 | g N m-2 | Rastetter et al. (2013) |
| 6 | *DCWP* | Coarse woody debris C | 0.51 | g P m-2 | Rastetter et al. (2013) |
| 7 | *DC1* | Phase I soil organic C | 3075 | g C m-2 | *DC1* plus DOC in Rastetter et al. (2013) |
| 8 | *DN1* | Phase I soil organic N | 134.65 | g N m-2 | *DN1* plus DON in Rastetter et al. (2013) |
| 9 | *DP1* | Phase I soil organic P | 6.830 | g P m-2 | Rastetter et al. (2013) |
| 10 | *DC2* | Phase II soil organic C | 12770 | g C m-2 | Rastetter et al. (2013) |
| 11 | *DN2* | Phase II soil organic N | 724.13 | g N m-2 | Rastetter et al. (2013) |
| 12 | *DP2* | Phase II soil organic P | 47.885 | g P m-2 | Rastetter et al. (2013) |
| 13 | *EN* | Inorganic N | 0.83 | g N m-2 | NH4 plus NO3 in Rastetter et al. (2013) |
| 14 | *EP* | Inorganic P | 0.081 | g P m-2 | Rastetter et al. (2013) |
| 15 | *VC* | C effort | 0.436 | fraction | *VC* = total canopy effort and *VN = VP* = 1/2 total root effort from Rastetter et al. (2013) |
| 15 | *VN* | N effort | 0.282 | fraction |
| 15 | *VP* | P effort | 0.282 | fraction |
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| **Allometry and Flux Values** | | | | | |
| Eq. | Symbol | Name | Value | Units | Comments |
| 16 | *BT* | Total biomass | 26680 | g DW m‑2 | Calculated internally |
| 17 | *BA* | Active biomass | 1025 | g DW m‑2 | Calculated internally |
| 18 | *BL* | Leaf biomass | 447 | g DW m‑2 | Calculated internally |
| 19 | *BR* | Fine-root biomass | 578 | g DW m‑2 | Calculated internally |
| 20 | *BW* | Woody biomass | 25655 | g DW m‑2 | Calculated internally |
| 21 | *qN* | Optimal biomass N concentration | 0.00302 | g N g-1 DW | Calculated internally |
| 22 | *qP* | Optimal biomass P concentration | 0.000290 | g P g-1 DW | Calculated internally |
| 23 | *UC* | Photosynthesis | 1230 | g C m-2 yr-1 | Rastetter et al. (2013) |
| 24 | *UN* | Plant N uptake | 13.51 | g N m-2 yr-1 | Total N uptake in Rastetter et al. (2013) |
| 25 | *UP* | Plant P uptake | 0.962 | g P m-2 yr-1 | Rastetter et al. (2013) |
| 26 | *Ra* | Autotrophic respiration | 509 | g C m-2 yr-1 | Maintenance plus NO3 uptake respiration in Rastetter et al. (2013) |
| 27 | *Rg* | Growth respiration | 158 | g C m-2 yr-1 | Rastetter et al. (2013) |
| 28 | *LC* | Fine litter C | 432 | g C m-2 yr-1 | Rastetter et al. (2013) |
| 29 | *LN* | Fine litter N | 12.93 | g N m-2 yr-1 | Rastetter et al. (2013) |
| 30 | *LP* | Fine litter P | 0.911 | g P m-2 yr-1 | Rastetter et al. (2013) |
| 31 | *LCWC* | Coarse litter C | 131 | g C m-2 yr-1 | Rastetter et al. (2013) |
| 32 | *LCWN* | Coarse litter N | 0.58 | g N m-2 yr-1 | Rastetter et al. (2013) |
| 33 | *LCWP* | Coarse litter P | 0.051 | g P m-2 yr-1 | Rastetter et al. (2013) |
| 34 | *RqC* | Plant C requirement | 982 | g C m-2 yr-1 | Calculated internally |
| 35 | *RqN* | Plant N requirement | 10.79 | g N m-2 yr-1 | Calculated internally |
| 36 | *RqP* | Plant P requirement | 0.768 | g P m-2 yr-1 | Calculated internally |
| 37 | *i* | Requirement: uptake ratio: for i = C, N, P | 0.832 | fraction | Calculated internally; all equal at steady state |
| 38 |  | Weighted geometric mean uptake: requirement ratio | 1.20 | fraction | Calculated internally |
| 39 | *MI* | Moisture index | 1 | none | At steady state assumed 1 |
| 40 | *TCWC* | Coarse debris turnover C | 131 | g C m-2 yr-1 | Rastetter et al. (2013) |
| 41 | *TCWN* | Coarse debris turnover N | 0.58 | g N m-2 yr-1 | Rastetter et al. (2013) |
| 42 | *TCWP* | Coarse debris turnover P | 0.051 | g P m-2 yr-1 | Rastetter et al. (2013) |
| 43 | *UNm* | Microbial N uptake | 54.04 | g N m-2 yr-1 | Rastetter et al. (2013) |
| 44 | *UPm* | Microbial P uptake | 7.720 | g P m-2 yr-1 | Rastetter et al. (2013) |
| 45 | *MC* | Microbial C use | 795 | g C m-2 yr-1 | Calculated internally |
| 46 | *MN* | Microbial N use | 80.84 | g N m-2 yr-1 | Calculated internally |
| 47 | *MP* | Microbial P use | 9.580 | g P m-2 yr-1 | Calculated internally |
| 48 | *C* | Microbial C efficiency | 0.443 | fraction | Calculated internally |
| 49 | *N* | Microbial N efficiency | 0.247 | fraction | Calculated internally |
| 50 | *P* | Microbial P efficiency | 0.138 | fraction | Calculated internally |
| 51 | *RCm1* | Phase I respiration | 443 | g C m-2 yr-1 | Rastetter et al. (2013) |
| 52 | *RNm1* | Phase I N mineralization | 60.87 | g N m-2 yr-1 | Rastetter et al. (2013) |
| 53 | *RPm1* | Phase I P mineralization | 8.260 | g P m-2 yr-1 | Rastetter et al. (2013) |
| 54 | *TC1* | Phase I to II C transfer | 113 | g C m-2 yr-1 | Rastetter et al. (2013) |
| 55 | *TN1* | Phase I to II N transfer | 6.39 | g N m-2 yr-1 | Rastetter et al. (2013) |
| 56 | *TP1* | Phase I to II P transfer | 0.422 | g P m-2 yr-1 | Rastetter et al. (2013) |
| 57 | *RCm2* | Phase II respiration | 113 | g C m-2 yr-1 | Rastetter et al. (2013) |
| 58 | *RNm2* | Phase II N mineralization | 6.39 | g N m-2 yr-1 | Rastetter et al. (2013) |
| 59 | *RPm2* | Phase II P mineralization | 0.422 | g P m-2 yr-1 | Rastetter et al. (2013) |
| 60 | *QN* | Inorganic N losses | 0.66 | g N m-2 yr-1 | NH4 and NO3 leaching plus denitrification in Rastetter et al. (2013) |
| 61 | *QP* | Inorganic P losses | 0.008 | g P m-2 yr-1 | PO4 leaching plus net 2o mineral formation in Rastetter et al. (2013) |
| 62 | *QDOC* | Organic C losses | 10 | g C m-2 yr-1 | Rastetter et al. (2013) |
| 63 | *QDON* | Organic N losses | 0.37 | g N m-2 yr-1 | Rastetter et al. (2013) |
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| **Driver Variables** | | | | | |
| Eq. | Symbol | Name | Value | Units | Comments |
| 7 | *IDOC* | Organic N inputs | 2 | g C m-2 yr-1 | Rastetter et al. (2013) |
| 8 | *IDON* | Organic N inputs | 0.08 | g N m-2 yr-1 | Rastetter et al. (2013) |
| 13 | *IN* | Inorganic N inputs | 0.95 | g N m-2 yr-1 | NH4 plus NO3 deposition in Rastetter et al. (2013) |
| 14 | *IP* | Inorganic P inputs | 0.008 | g P m-2 yr-1 | PO4 deposition plus 1o mineral weathering in Rastetter et al. (2013) |
| 23 | *I* | Solar radiation | 1000 | mol m-2 sec-1 | Assumed representative value to which model was calibrated |
|  | | | | | |
| **Parameters** | | | | | |
| Eq. | Symbol | Name | Value | Units | Comments |
| 15 | *a* | Acclimation rate | 1.1 | yr-1 | Rastetter et al. (2013) |
| 16 | *qC* | biomass C:dry weight | 0.45 | g C g-1 DW | Rastetter et al. (2013) |
| 17 | *BAx* | Maximum active biomass | 1110 | g DW m-2 | Rastetter et al. (2013) |
| 17 |  | Active:total at low biomass | 0.5 | fraction | Rastetter et al. (2013) |
| 21 | *qNL* | Leaf N concentration | 0.0221 | g N g-1 DW | Rastetter et al. (2013) |
| 21 | *qNW* | Woody tissue N concentration | 0.00227 | g N g-1 DW | Rastetter et al. (2013) |
| 21 | *qNR* | Fine-root N concentration | 0.02 | g N g-1 DW | Rastetter et al. (2013) |
| 22 | *qPA* | Leaf P concentration | 0.00173 | g P g-1 DW | Rastetter et al. (2013) |
| 22 | *qPW* | Woody tissue P concentration | 0.000226 | g P g-1 DW | Rastetter et al. (2013) |
| 22 | *qPR* | Fine-root P concentration | 0.002 | g P g-1 DW | Rastetter et al. (2013) |
| 23 | *SC* | sec to annual scalar | 27.99 | sec g C mol-1 yr-1 | Calibrated to *UC* |
| 23 | *Px* | Maximum photosynthesis | 35 | mol m-2 sec-1 | McMurtrie et al (1992) |
| 23 | *E0* | Quantum yield | 0.06 | mol mol-1 |
| 23 | *kI* | Beer's constant | 0.5 | m2 m-2 | Rastetter et al. (2013) |
| 23 | *LMA* | Leaf mass per area | 70.92 | m2 g-1 DW | Inverse of specific leaf area in Rastetter et al. (2013) |
| 24 | *UNx* | Maximum N uptake | 0.1031 | g N g-1 DW yr-1 | Calibrated to *UN* |
| 24 | *kN* | Plant N 1/2 saturation constant | 1 | g N m-2 | Calibrated to the bole-only response in Rastetter et al. (2013) |
| 25 | *UPx* | Maximum P uptake | 0.007521 | g P g-1 DW yr-1 | Calibrated to *UN* |
| 25 | *kP* | Plant P 1/2 saturation constant | 0.102 | g P m-2 | Calibrated to the bole-only response in Rastetter et al. (2013) |
| 26 | *ra* | Metabolic respiration constant | 6.39 | g C g-1 N yr-1 | Calibrated to *Ra* |
| 27 | *rg* | Growth respiration constant | 0.281 | g C g-1 C | Calibrated to *Rg* |
| 28 | *tL* | Leaf turnover | 0.866 | yr-1 | Weighted mean of evergreen and deciduous rates in Rastetter et al. (2013) scaled to annual |
| 28 | *tW* | Woody biomass turnover | 0.0112 | yr-1 | Calibrated to *LC* |
| 28 | *tR* | Fine-root turnover | 0.493 | yr-1 | Rastetter et al. (2013) scaled to annual |
| 29 | *qlLN* | Leaf litter N | 0.0124 | g N g-1 DW | Calibrated to *LN*, maintaining same relative magnitudes as in Rastetter et al. (2013) |
| 29 | *qlWN* | Woody litter N | 0.00595 | g N g-1 DW |
| 29 | *qlRN* | Root litter N | 0.0220 | g N g-1 DW |
| 30 | *qlLP* | Leaf litter P | 6.74x10-4 | g P g-1 DW | Calibrated to *LP*, maintaining same relative magnitudes as in Rastetter et al. (2013) |
| 30 | *qlWP* | Woody litter P | 4.19x10-4 | g P g-1 DW |
| 30 | *qlRP* | Fine-root litter P | 9.20x10-4 | g P g-1 DW |
| 31 | *tCW* | Coarse woody biomass turnover | 0.0113 | yr-1 | Calibrated to *LCWC* |
| 31 | *mCWX* | Canopy closure parameters | 0.0548 | yr-1 | Rastetter et al. (2013); mCWX scaled to annual value |
| 31 | *kWL* | 1x10-7 | m4 g-2 C |
| 31 | *BWc* | 8000 | g DW m-2 |
| 32 | *qlCWN* | C:N coarse woody debris | 226 | g C g-1 N | = *DCWC/DCWN* |
| 33 | *qlCWP* | C:P coarse woody debris | 2575 | g C g-1 P | = *DCWC/DCWP* |
| 34 | *gC* | Gain for C | 0.6 | none | Calibrated to steady state for *VC, VN,*  and *VP* |
| 35 | *gN* | Gain for N | 0.803 | none |
| 36 | *gP* | Gain for P | 0.950 | none |
| 39 | *aMI* | Moisture index parameters | 1.30 | none | Calibrated to the bole-only response in Rastetter et al. (2013) |
| 39 | *bMI* | 2x10-7 | m4 yr2 g-4 C |
| 40 | *rCWC* | Coarse woody debris turnover | 0.0998 | yr-1 | Calibrated to *TCWC* |
| 43 | ** | N immobilization constant | 0.0626 | g N g-1 C yr-1 | Calibrated to *UNm* |
| 43 | *N* | Soil C:N limit | 17.6 | g C g-1 N | = *DC2 / DN2* |
| 43 | *kNm* | Microbial N 1/2 saturation constant | 3 | g N m-2 | Calibrated to the bole-only dynamics in Rastetter et al. (2013) |
| 44 | *P* | P immobilization constant | 0.0290 | g P g-1 C yr-1 | Calibrated to *UPm* |
| 44 | *P* | Soil C:P limit | 267 | g C g-1 P | = *DC2 /DP2* |
| 44 | *kPm* | Microbial P 1/2 saturation constant | 1.5 | g P m-2 | Calibrated to the bole-only dynamics in Rastetter et al. (2013) |
| 45 | *C* | Organic C use constant | 0.0763 | yr-1 | Simultaneously calibrated to *RNm, RPm,*and *RPm* |
| 46 | *N* | Organic N use constant | 0.0434 | yr-1 |
| 47 | *P* | Organic P use constant | 0.272 | yr-1 |
| 45 | *CN* | Dissolved organic matter (DOM) use constant | 0.00135 | m2 g-1 N yr-1 | Calibrated to the bole-only dynamics in Rastetter et al. (2013) |
| 46 | *qDOM* | C:N of DOM | 26.72 | g C g-1 N | Rastetter et al. (2013) |
| 48 | *C* | Maximum microbial efficiency | 0.72 | fraction | Calibrated to the bole-only dynamics in Rastetter et al. (2013) |
| 54 | *rT* | Phase I to II conversion rate | 0.0366 | yr-1 | Calibrated to *TC1* |
| 57 | *rm2* | Phase II turnover constant | 0.00882 | yr-1 | Calibrated to *RCm2* |
| 60 | *N* | N loss constant | 0.790 | yr-1 | Calibrated to *QN* |
| 61 | *P* | P loss constant | 0.0926 | yr-1 | Calibrated to *QP* |
| 62 | *DOM* | DOM loss constant | 2.36x10-5 | yr-1 | Calibrated to *QDOC* |

Citations:

McMurtrie, RE, HN Comins, MUF Kirschbaum, and Y-P Wang. 1992. Modifying existing forest growth models to take account of effects of elevated CO2. Australian J. Botany 40:657-677.

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